



# 1 Introduction

This paper follows a number of recent studies in focusing on ex-ante real yields derived from inflation-indexed bonds. Given that standard asset pricing theory ascribes changes









96.3% and 98.18% of the variance, respectively. Obviously, three factors are not going to be enough to explain the combined term structure. On the other hand, five factors can explain as much variability as three factors can for the real and nominal parts separately. Nevertheless, I chose to model the combined term structure with four factors so to favor the parsimony of the model and avoid overfitting.

### **3.2 Liquidity in the Israeli Government Bond Market**

Several studies of the U.S. market, such as Sack and Elsassner (2002), Shen (2006), and D'Amico et al. (2018), conclude that prior to 2004, TIPS yields were too high. This suggests that there might have been a significant liquidity premium that has since declined. A recent paper by Fleckenstein, Longstaff, and Lustig (2014) shows that there are arbitrage opportunities in the TIPS market that were economically significant particularly during

on weekly 3-month, 2-year, 10-year nominal yields should result in a high  $R^2$ . The results (untabulated in the paper) show an  $R^2$  of 87% for the whole period, much higher than the 6% reported in D'Amico et al. (2018). These results suggest that the variation in the real yields implied from inflation-indexed bonds is due to variation in the "true" real yields and





where  $\mathbf{f}$  is a  $(4 \times 1)$  vector,  $\mathbf{A}$  is a  $(4 \times 4)$  matrix,  $\mathbf{f}e_tg$  is a  $(4$



Assuming that the market is complete implies that

$$\frac{M_{t+1}^R}{M_t^R} = \frac{M_{t+1}^N}{M_t^N}$$



Expected  $n$ -month ahead short rates are easily derived and take the form<sup>18</sup>

$$E_t(r_{t+n}^N) = \frac{N}{0} + \frac{N^0}{1} \dots X_t. \quad (17)$$

I assume that one-month and one-year ahead mean survey forecasts have no bias in their expectations, i.e.,

$$\begin{aligned} I_t^2 &= E_t(r_{t+1}^N) + i_2, \\ I_t^{13} &= E_t(r_{t+12}^N) + i_{13}, \end{aligned} \quad (18)$$

and

$$\begin{matrix} & O & 1 \\ I^N & @ & i_2 \\ & 0 & i_{13} \end{matrix} \begin{matrix} \\ \\ C \\ A \end{matrix} : \quad (19)$$

A disadvantage of the Israeli data is that there are no long-term surveys. This may

## 5.4 The Augmented State and Measurement Equation

To x ideas, I conclude this section with a more explicit representation and the identification

The augmented measurement equation takes the form

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$y_t = Bx_t + v_t$





significance of only 10%, supporting the notion that part of the fitting error may be due to seasonality noise.

## 6.2 Decomposing the Yield Curve

described by the two models are not corroborated by the Israeli data. Our results are however in line with the model of Wachter (2006) who develops a consumption-based model of the term structure of interest rates. She calibrates her model so that the real risk-free rate is negatively correlated with surplus consumption. The negative correlation between surplus consumption and the risk-free rate leads to positive risk premia on real bonds, and an upward-sloping yield curve.

10-year maturity.

It is interesting to compare these results with other/

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interesting to identify the factors, as macro- finance models do. I leave this to future work.



Assume that we can observe only the multivariate series  $f$  (or  $g$ ) when what we really want to know is the dynamics of the whole system  $x$ . Suppose that we have a model of the system  $x$  in the form of a state space model (SSM)  $(A, B, C, D)$  where  $A$  is the state transition matrix,  $B$  is the input matrix,  $C$  is the output matrix, and  $D$  is the direct transmission matrix. The state space model is defined by the following equations:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

where  $x$  is the state vector,  $u$  is the input vector, and  $y$  is the output vector. The state vector  $x$  is of dimension  $n$ , the input vector  $u$  is of dimension  $m$ , and the output vector  $y$  is of dimension  $p$ . The state transition matrix  $A$  is of dimension  $n \times n$ , the input matrix  $B$  is of dimension  $n \times m$ , the output matrix  $C$  is of dimension  $p \times n$ , and the direct transmission matrix  $D$  is of dimension  $p \times m$ .

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The main takeaway is to note that the marginal distribution of  $y_t$  is

$$f(j|y_{tjt-1}) = N(y_{tjt-1})$$



## B Deriving the Pricing Equations

Note that

$$P_t^1 = E(M_{t+1}) = \exp(-r_t) = \exp(-r)$$

## References

- Michael Abrahams, Tobias Adrian, Richard K. Crump, Emanuel Moench, and Rui Yu. Decomposing real and nominal yield curves. *Journal of Monetary Economics*, 84:182{200, 2016.
- Geert Bekaert and Xiaozheng Wang. Inflation risk and the inflation risk premium. *Economic Policy*, 25(64):755{806, 2010.
- John Y. Campbell and Robert J. Shiller. Yield spreads and interest rate movements: A bird's eye view. *The Review of Economic Studies*, 58(3):495{514, 1991.
- Qiang Dai and Kenneth J Singleton. Specification analysis of affine term structure models. *The Journal of Finance*, 55(5):1943{1978, 2000.
- Qiang Dai and Kenneth J. Singleton. Expectation puzzles, time-varying risk premia, and affine models of the term structure. *Journal of Financial Economics*, 63(3):415{441, 2002.
- Stefania D'Amico, Don H. Kim, and Min Wei. Tips from TIPS: The informational content of treasury inflation-protected security prices. *Journal of Financial and Quantitative Analysis*, 53(1):395{436, 2018.

Eugene F. Fama and Robert R. Bliss. The information in long-maturity forward rates.

Emi Nakamura, Jon Steinsson, Robert Barro, and Jose Ursua. Crises and recoveries in an empirical model of consumption disasters.

Jessica A. Wachter. A consumption-based model of the term structure of interest rates.  
*Journal of Financial Economics*

**Table 1: Summary Statistics.** This table shows summary statistics of nominal and real yields with maturities of one, three, five, seven, and ten. The frequency of the data is monthly (end-of-month) and it spans the period of 01/1985{03/2018 for real yields and the period of 05/2001{2018 for nominal yields. From 05/2001 onwards, the yields are of constant maturity and smoothed. Before that, yield to maturity of inflation-indexed bonds are used. Real yields are adjusted for both the effect of carry and seasonality. Data is in percent and annualized. Yields are continuously compounded.

(a) Real Yields 01/1985{03/2018

Maturity	1	3	5	7	10
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**Table 2: Fit Statistics.** This table shows the RMSE and the MAD of the estimated yields, both real and nominal. Data is in basis points.

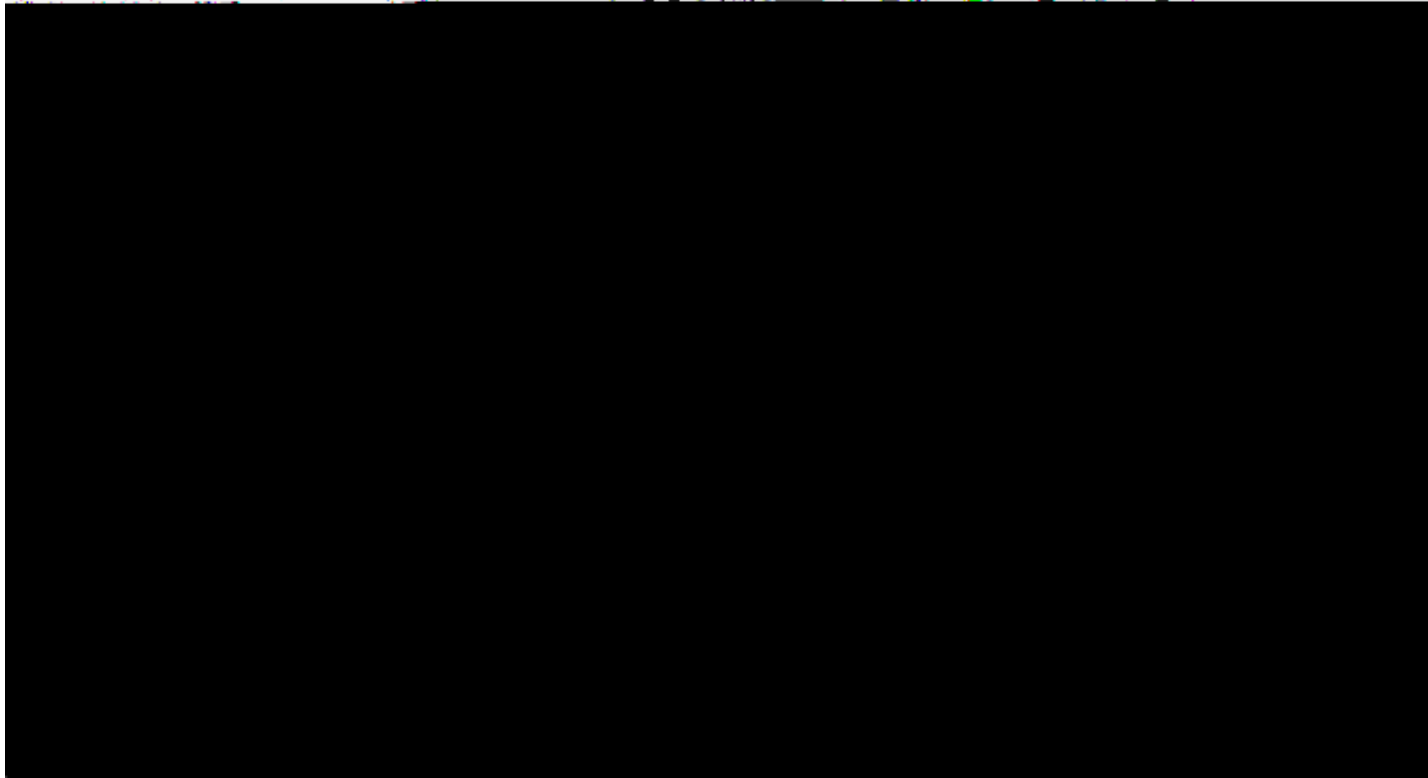
(a) Real rates (01/1985{03/2018)

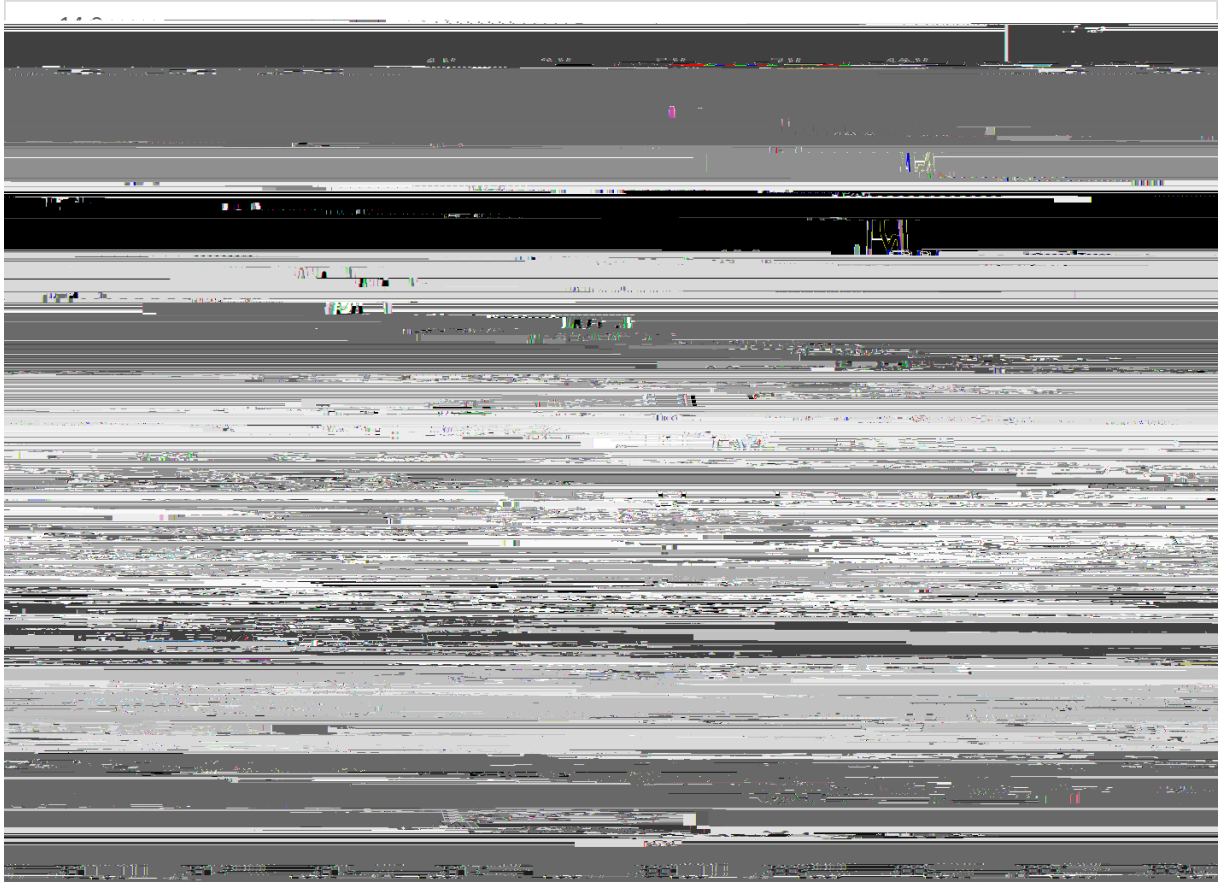






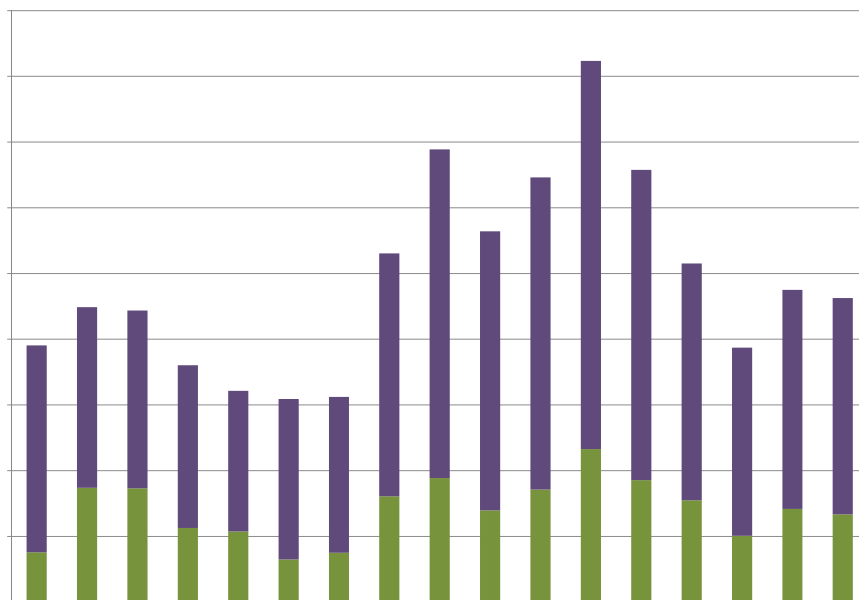
Figure 1 Time Series of Real Yields



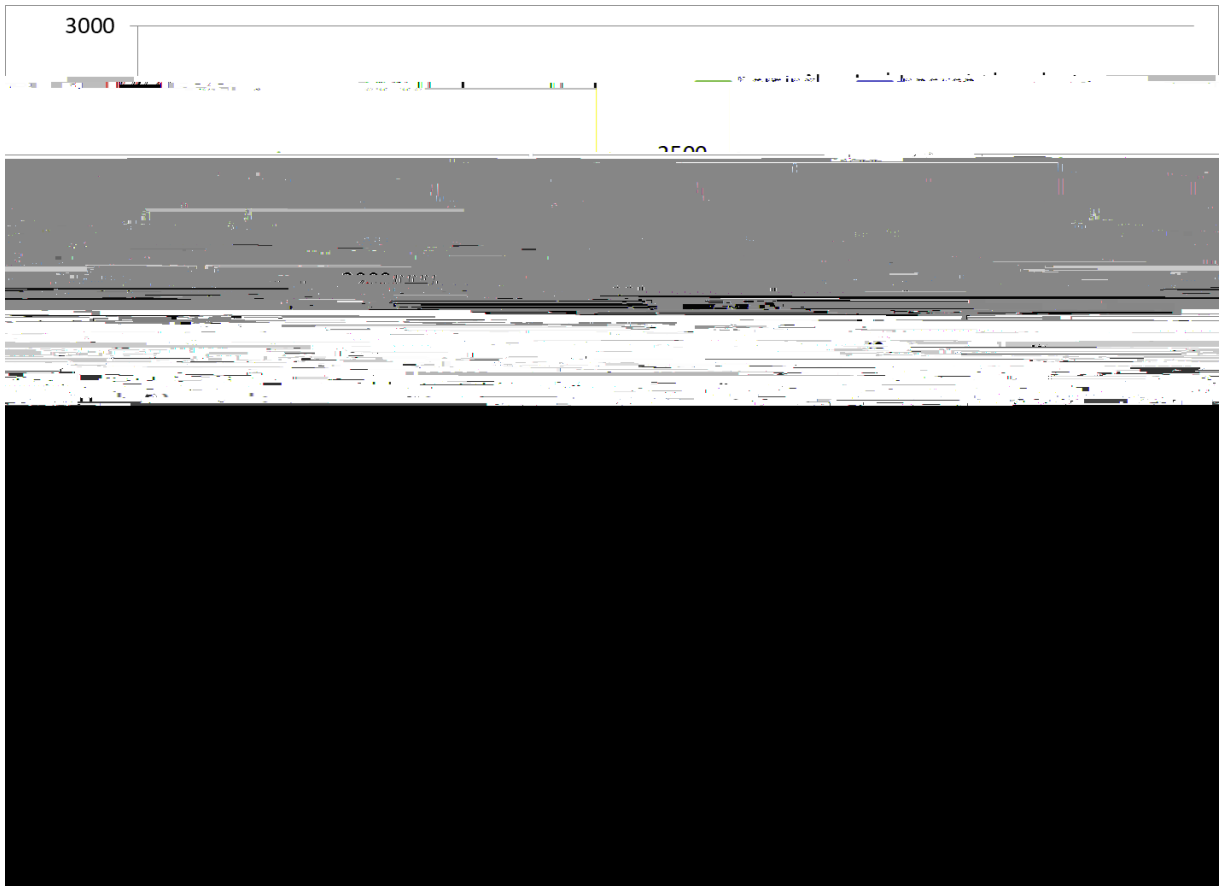


**Figure 2: Time Series of Nominal Yields.** This figure presents the time series of nominal yields with maturities of one, three, five, seven, and ten years used in the estimation of the model. Data is monthly (end-of-month) and yields are in percent annualized.



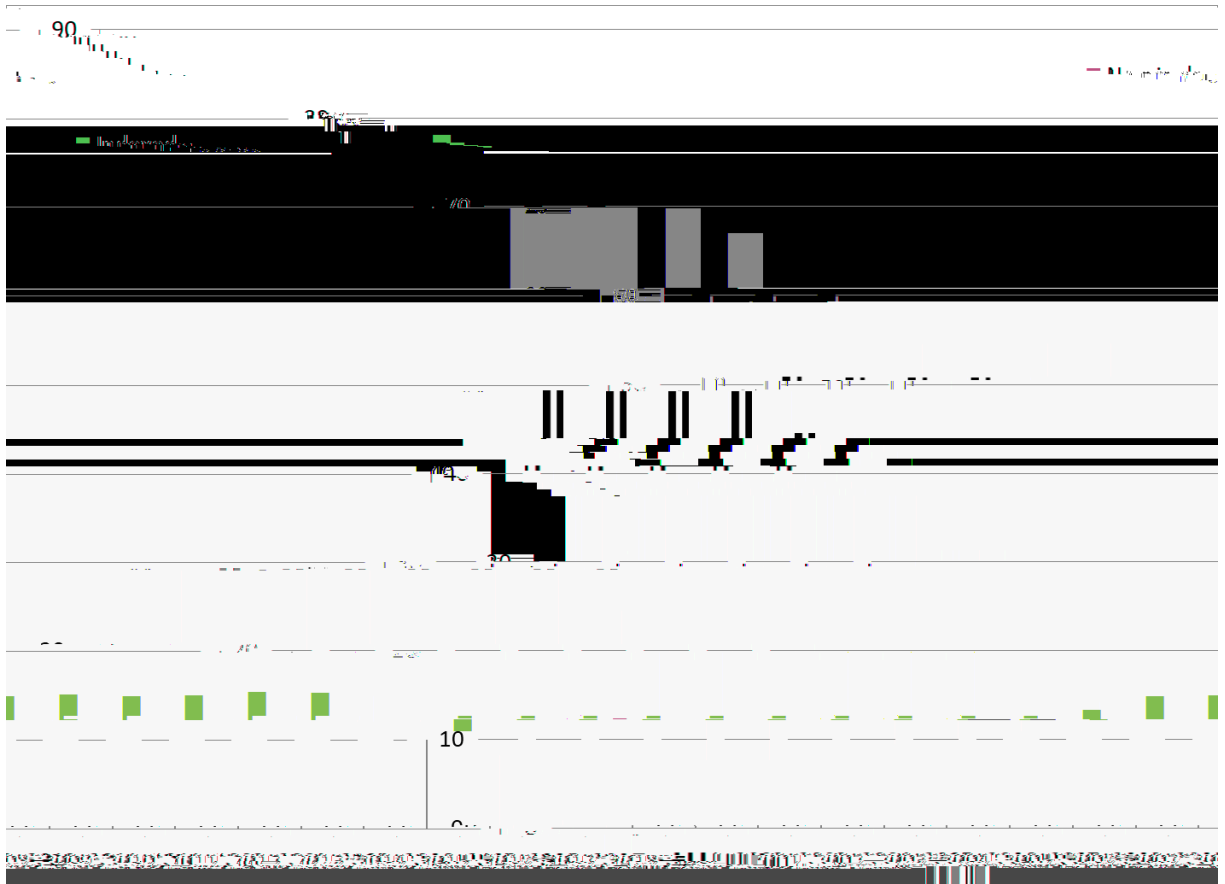


**Figure 4: Issuance of indexed and non-indexed bonds.** This figure plots yearly issuance of indexed and nominal bonds. Data is in millions of NIS.



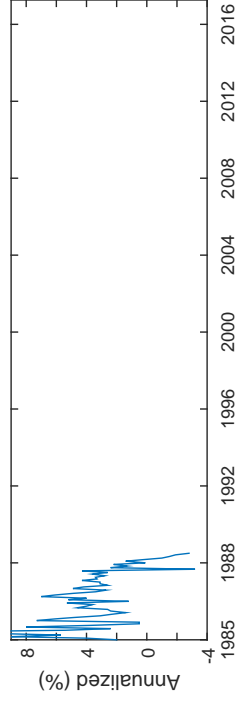
**Figure 5: Volume** (in millions of NIS) of bonds. The figure plots the trading volume of indexed and non-indexed bonds in the Israeli stock market.





**Figure 7: Number of Series.** This figure plots the number of series of Indexed and non-indexed bonds.





**Figure 8: Real Yields Comparison.** This figure plots fitted real yields from the three factor model and actual real yields for maturities of 1,3,5,7 and 10 years.

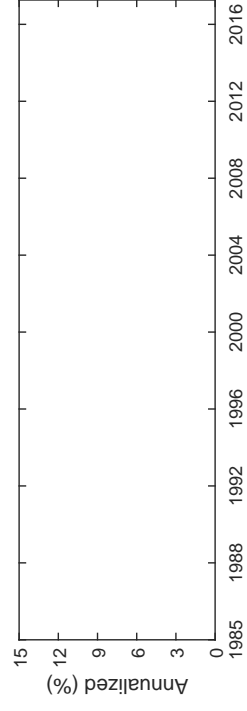


Figure 9: Nominal Yields Comparison. This figure plots fitted nominal yields from the three factor model and actual





Figure 11: World Inflation. The figure plots the annual CPI inflation for selected countries.

